National Technical University of Ukraine
"Igor Sikorsky Kyiv Polytechnic Institute"

Department of
Biomedical Engineering

HIGHER MATHEMATICS

Work program of the discipline (Syllabus)

Details of the discipline	
Level of higher education	***First (bachelor's)***
Branch of knowledge	*16 Chemical and bioengineering*
Specialty	*163 Biomedical Engineering*
Educational program	*Medical Engineering*
Discipline status	*Normative*
Form of study	*full-time / day / remote*
Year of preparation, semester	*1-2-nd years, three semesters*
The scope of discipline	20,5 *ECTS credits (1 semesters 72 hours - lectures, 72 hours - practical, 126 hours - self-study, 2 semesters 54 hours - lectures, 36 hours - practical, 60 hours - self-study, 3 semesters 54 hours - lectures, 72 hours - practical, 69 hours – self-study)*
Semester control / control measures	*Exams, MCR, CGW*
Lessons schedule	*According to the schedule on the website http://rozklad.kpi.ua/*
Language of instruction	*English*
Info for erivnyk and course / coaches in	*Lecturer:* PhD, Senior Lecturer, Syrotenko Anton, mobile +380986680182, email: antonsyrotenko86@gmail.com
Course placement	*Sikorsky Platform, Zoom*

The program of discipline

1. Description of the discipline, its purpose, subject of study and learning outcomes

The discipline "Higher Mathematics" plays a significant role in the preparation of bachelors in the specialty 163 "Biomedical Engineering". Study subjects promote analytic thinking and allow the use of approaches, methods, and knowledge of mathematics at mastering other disciplines of special profile, lays the foundation for professional competence.

The discipline studies the basic concepts and theorems of mathematic analysis, linear algebra and analytic geometry and their consequences; apply methods of differential calculus of a function of one and many variables; use methods of integral calculus of a function of one and many variables to solve geometric, physical, mechanical and other problems; apply the basics of Field Theory to understand dynamic processes; understand the possibility of using differential equations in solving practical problems; understand the possibility of using numerical and functional series in approximate calculations and process modeling; use methods and theorems of the theory of functions of a complex variable and operational

calculus in search of optimal models for biological systems; methods of application of the theoretical apparatus of mathematic in solving practical problems of biomedical engineering.

The purpose of discipline is to develop in students competencies in line with the educational professional program "Medical Engineering" (order NON / 89/2021 of 04/19/2021):

General competencies

GC 1	Ability to apply knowledge in practical situations.
GC 3	Ability to communicate in the state language both orally and in writing.
GC 4	Skills in the use of information and communication technologies.
GC 5	Ability to perform research at the appropriate level.
GC 6	Ability to search, process and analyze information from various sources.
GC 7	Ability to generate new ideas (creativity).
GC 8	Ability to make well-grounded decisions.
GC 9	Ability to communicate with representatives of other professional groups of different levels (with experts from other fields of knowledge / types of economic activity).
GC 10	Safe activities skills.

Professional competencies

PC 5	Ability to apply mathematical methods in the analysis, modeling of the functioning of living organisms and biotechnical systems.

Program learning outcomes

PLO 1	Ability to use engineering software packages for research, analysis, processing and presentation of results, as well as for automated design of medical devices and systems.
PLO 24	Apply knowledge of the basics of mathematics, physics and biophysics, bioengineering, chemistry, engineering graphics, mechanics, resistance and strength of materials, properties of gases and liquids, electronics, computer science, obtaining and analyzing signals and images, automatic control, systems analysis and decision of making methods at the level required to solve the problems of biomedical engineering.

According to the requirements of the program of the discipline"Higher Mathematics", students must demonstrate the following learning outcomes:

knowledge:

- Basic definitions of the functions of one and many variables (area of definition, methods for specifying functions, graphs and properties of elementary functions);
- Limits of numerical sequences, limits of functions at a point, the first and second important limits, the comparison of infinitesimal and properties of equivalent functions, the continuity of functions at the point, the discontinuities of functions;
- The basis of the differential calculus, the derivative and the differential of the functions of one and many variables, the tangent and normal to the curve, the tangent plane and the normal to the surface, the extremums of functions, the Lopital rule;

- Ability to operate on the properties of an indefinite, definite and improper integral of a function of one variable; apply integrals to solve geometric, mechanical, physical and other problems.
- Find the areas, volumes, masses and charges of one-dimensional, two-dimensional and three-dimensional objects using multiple and linear integrals and investigate their basic mechanical properties (center of mass, moments of inertia, static moments);
- Use methods and theorems of Field Theory in search of optimal models for biological systems.
- Use knowledge of methods for solving certain types of differential equations and systems of differential equations.
- Basic properties of numerical and functional series in approximate calculations and representation of functions in the form of Taylor's Series and Maclaurin's Series and series and integral Fourier of periodic and nonperiodic functions;
- Use methods and theorems of the theory of functions of complex variable and operational calculus in search of optimal models for biological systems.
- Be able to apply scientific predictions, forms and methods of analysis, processing and synthesis of information using basic knowledge of probability theory.

skills:

- Apply methods of differential calculus of a function of one variable;
- Apply methods of differential calculus of the function of many variables;
- Use methods of integral calculus of a function of one variable to solve geometric, physical, mechanical and other problems
- Use methods of integral calculus of the function of many variables to solve geometric, physical, mechanical and other problems;
- Understand the possibility of using differential equations in solving practical problems;
- Understand the possibility of using numerical and functional series in approximate calculations and process modeling;
- Use methods of operational calculus;
- Understand the possibility of using basic knowledge of probability theory.

experience:

- Learn how to work with information resources, textbooks, reference books, etc.;
- To master the methods of mathematical analysis for solving problems of corresponding sections of higher mathematics;
- Learn how to solve technical problems obtained as a result of mathematical modeling of processes.

2. Prerequisites and postrequisites of the discipline (place in the structural-logical scheme of education according to the relevant educational program)

The discipline "Higher mathematics", belongs to the normative disciplines of the cycle of professional training and is based on knowledge of the disciplines: "Mechanics", "Physics-1", "Engineering and computer graphics", "Materials science and construction materials".

Theoretical knowledge and practical skills acquired during the study of the discipline "Higher mathematics" are used in mastering the following disciplines: "Biomedical mechanics",

"Engineering mechanics", "Therapeutic medical equipment", "Development and operation of physiotherapeutic medical devices".

A general course in "Higher Mathematics" is the foundation of a specialist in mathematical and engineering education. Mathematical research methods penetrate into all areas of human activity, and because of this, there is a growing interest in the general course of higher mathematics from the side of related sciences, which use a different volume of mathematical knowledge.

3. The content of the discipline

The main tasks of the cycle of lectures and practical classes: to learn to apply theoretical knowledge to solving practical problems, to develop the skills of working with information resources and to master the methods of mathematical analysis and analytical geometry for solving problems arising as a result of modelling of technical processes.

List of sections and topics of the whole discipline

- **Matrices and operations with them.** The inverse matrix and its properties. Matrix equations.
- **System of equations**. Inhomogeneous and homogeneous system of equations.
- **Vectors and linear operations with them.** Scalar projection. Unit vector. Components of a vector. Direction cosines. Magnitude of a vector. Angle between two vectors. Scalar product.
- **Vector product. Scalar triple product**. Their geometrical interpretations, properties and applications.
- **Lines in a plane.** Different problems with lines in a plane.
- **Plane.** Different forms of the equation of a plane. Relationship between lines, the angle between two lines, the shortest distance from a point to a line.
- **Lines in 3D. Line and plane.** Condition under which two lines are coplanar. Lines and planes combined in 3D. Angle between a line and a plane. Intersection of a line and a plane. Different problems for lines and planes in 3D.
- **Curves of the Second Order in Plane.** Parabola. Ellipse. Hyperbola.
- **Curves of the Second Order in 3D.** Sphere. Cone. Paraboloid. Cylinder.
- **Sylvester's criterion.**
- **The basis of the differential calculus**: the derivative and the differential of the functions of one and many variables, the tangent and normal to the curve, the tangent plane and the normal to the surface, the extremums of functions of one and many variables.
- **Definition of a definite integral.**
- Properties and calculation of a definite integral. **Newton's – Leibniz formula**. Improper integrals. Application of a definite integral.
- **Integral calculus of functions of many variables. Double and triple integrals.** Definitions, properties, calculations and applications. Replacement of variables in multiple integrals. Jacobian transition.
- **Curvilinear integrals** of the first and second kind: definition, calculation and application.
- **Surface integrals** of the first and second kind: definition, calculation and application. Green's Formula. Stokes' Formula. Ostrogradsky's Formula. The Hamiltonian Operator and Certain Applications of It.

- **Ordinary differential equations.** Problems leading to the concept of DE. First-order differential equations with separable variables, homogeneous, linear, Bernoulli equations. **Cauchy's problem (IVP)**. Equations that allow lowering the order, **LDE with constant coefficients, special and general right-hand sides.** DE systems.
- **Numerical Series**. Sum of a Numerical Series. Converges and diverges of Series. Necessary Condition for Convergence of a Numerical Series. Sufficient condition for convergence. **Alternating Series.** Leibniz' Theorem. **Power Series.** Interval of Convergence. Integration and Differentiation of Series. Power and Numerical Series for Complex domain. **Taylor's Series and Maclaurin's Series.** Euler's Formula. **Fourier series.** Fourier Series for periodic function $f(x)$ with period 2π. Fourier Series for Even and Odd Functions. The Fourier Series for a Function with Period 2l. On the Expansion of a Nonperiodic Function in a Fourier Series.**Fourier Integral.** The Fourier Integral in Complex Form.
- **Introduction to Probability Theory.** Random Experiments and Probability Models. Definitions. Product Rule. Law of Total Probability and Bayes' Rule. Independence.
- **Random Variables and Probability Distributions.** Random Variables. Probability Distribution. Expectation. Some Important Discrete and Continuous Distributions.
- **Generating Random Variables on a Computer.**
- **Joint Distribution.** Joint Distribution and Independence. Discrete and Continuous Joint Distributions Expectation. Conditional Distribution.
- **Functions of Random Variables and Limit Theorems.** Jointly Normal Random Variables.

4. Training materials and resources

Basic literature:

1. N. Piskunov. Differential and integral calculus.-M.: MIR Publishers., 1969.

   https://pisethsok.files.wordpress.com/2017/11/differential-and-integral-calculus-n-piskunov-editado.pdf

2. Differential calculus of function of one variable.-Repeta Lesya, Mulyk Olena
   https://campus.kpi.ua/tutor/index.php?mode=mob&show&irid=217758

3. Functions of Several Variables \ Arindama Singh\ Department of Mathematics Indian Institute of Technology Madras
   https://campus.kpi.ua/tutor/index.php?mode=mob&show&irid=216191
4. Differential and Integral calculus with examples and applications/George A. Osborne, S.B. – walker professor of mathematics in the Massachusetts instinute of technology. – Boston, USA D.C. Heath & Co Publishers, 1906.
5. Берман Г. Н. Сборник задач по курсу математического анализа / Г. Н. Берман. — С.Пб.: Лань, Специальная литература, 2002. — 448 с. — ISBN 5- 8114-0107-8.
6. Analytic Geometry in Two and Three Dimensions.
   https://vonsteuben.enschool.org/ourpages/auto/2013/8/26/58023530/Chapter%208%20_pg_%20631-698_.pdf
7. Applied Engineering Analysis - slides for class teaching * Chapter 4 Linear Algebra and Matrices\ Based on the book of "Applied Engineering Analysis", by Tai-Ran Hsu,

published by John Wiley & Sons, 2018. (ISBN 9781119071204)\
https://www.sjsu.edu/me/docs/hsu-Chapter%204%20Linear%20Algebra%20and%20Matrices.pdf
8. A Short Introduction to Probability\Prof. Dirk P. Kroese School of Mathematics and Physics The University of Queensland\2018 D.P. Kroese\
https://people.smp.uq.edu.au/DirkKroese/asitp.pdf

Additional literature:

1. Understanding Basic Calculus/ S.K. Chung /Department of Mathematics, University of Hong Kong http://math.schung.info/calculus/basic_calculus.html.
2. Диференціальне та інтегральне числення функцій однієї змінної. Практикум. (І курс І семестр) / Уклад.: І. В. Алєксєєва, В. О. Гайдей, О. О. Диховичний, Л. Б. Федорова. — К: НТУУ «КПІ», 2013. — 252 с.
3. Диференціальне та інтегральне числення функцій однієї змінної. Конспект лекцій. (І курс І семестр) / В. О. Гайдей, Л. Б. Федорова, І. В. Алєксєєва, О. О. Диховичний. — К: НТУУ «КПІ», 2013. — 104 с.
4. Письменный Д. Конспект лекций по высшей математике. Полный курс / Д. Письменный. — М.: Айрис-Пресс, 2008. — 608 с. ISBN 978-5-8112-3118-8, 978-5-8112-3480-6.
5. Limits and continuity – A guide for teachers (Years 11–12) Principal author: Peter Brown, University of NSW/ Australian Mathematical Sciences Institute Building 161 The University of Melbourne VIC 3010 Email: enquiries@amsi.org.au Website: www.amsi.org.au
6. Math 221 first semester calculus\2010\The Free Lecture Notes Page (wisc.edu)
7. The Derivative\2010\The Free Lecture Notes Page (wisc.edu)
8. Calculus with analytic geometry second edition George F. Simmons Colorado College, Colorado Springs\ ISBN 0-07-057642-4\1996 http://www.ru.ac.bd/wp-content/uploads/sites/25/2019/03/205_03_Simmons_Calculus-with-analytic-geometry.pdf

Educational content

5. Methods of mastering the discipline (educational component)

The discipline **"Higher Mathematics-1"** consists of two interrelated parts, which are studied in parallel in the first autumn semester, so the educational component is presented in different tables.

MATHEMATICAL ANALYSIS

Names of sections and topics	Program learning outcomes	The main tasks	
		Learning process	Deadline
Section 1. Number. Variable. Function. Limit. Continuity of a function.			
Topic 1.1. **Introduction to mathematical analysis.** The limit of the numerical sequence. Infinitely small and infinitely large sequences. Number ***e.***	PLO 1, 24	**Lecture 1** Classroom work 1 Classroom work 2	1st week
Topic 1.2. **The limit of the function.** Definition of the limit of the function in Heine and Cauchy. Conditions of the existence of the limit of the function. Theorems on the limit of the function	PLO 1, 24	**Lecture 2** **Lecture 3** Classroom work 3	3nd week
Topic 1.3. **First and second important limits.** Comparison of infinitely small functions. Comparison of infinitesimal functions, Applying equivalences to calculating limits of functions	PLO 1, 24	**Lecture 4** Classroom work 4	4th week
Topic 1.4. **Continuity of the function in the point.** Theorems on continuous functions in a point. Types of discontinuity. Breakpoints, their types. Classification of breakpoints. **Continuity of the function on the segment.** Theorems on continuous functions on a segment: Weierstrass, Cauchy, Bolzano-Cauchy. Theorem on the continuity of an inverse function.	PLO 1, 24	**Lecture** 5 Classroom work 5	5th week
Section 2. Derivative and differential			
Topic 2.1. **The derivative.**The definition of the derivative. Definition of the derivative at the point. Geometric, physical content of the derivative. The tangent and normal to the function graph. **Differentiation of functions.** Derivative of inverse, composite functions, composite exponential function, functions given implicitly and parametrically. Derivative of power-index function.	PLO 1, 24	**Lecture 6** Classroom work 6 **Lecture 7** Classroom work 7	6th week 7th week

Topic 2.2. **Derivatives of higher orders.** Derivatives of higher orders of functions given explicitly, implicitly, and parametrically. Formula Leibniz.	PLO 1, 24	**Lecture 8** Classroom work 8	8th week
Topic 2.3. **Differential of the functions.** The meaning of differential and geometric meaning. Properties of the differential. **Differentiability of the function.** Differentials of higher orders. Non-invariance of the form of the second differential. **Basic theorems of differential calculus.** Theorems of Lagrange, Role, Cauchy. **Rule of Lopital.** Application of Lopital`s rule.	PLO 1, 24	**Lecture 9** Classroom work 9 **Lecture 10**	9th week
Topic 2.4. **Taylor's formula.** Remainder of the Taylor formula in the form of Lagrange, Peano. Applying the Taylor formula to calculating approximate values and boundary functions. **Investigation of the function.** Local extremums of the function.	PLO 1, 24	**Lecture 11** Classroom work 10 **Lecture 12** Classroom work 11	10th week 11th week
Topic 2.5. **Convex functions.** Necessary and sufficient conditions of the convexity of the function on the interval. The points of the bend, sufficient conditions for the existence of the point of inflection. **Graphing a function**. Asymptotes.	PLO 1, 24	**Lecture 13** Classroom work 12	12th week
Modular control work		Writing MCR / testing	13th week
Section 3. **Functions of several variables**			
Topic 3.1. **Differentiation of functions of many variables.** Partial derivatives. Full increment and complete differential of functions, its connection with partial derivatives. The geometiuc interpretation of the partial derivatives of a function of two variables.	PLO 1, 24	**Lecture 15** Classroom work 14	14th week
Topic 3.2. **Derivatives of higher orders of functions of many**	PLO 1, 24	**Lecture 16** **Lecture 17**	15th week

variables. Differentiability of the composite function. Partial derivatives and full differentials of higher orders. **Directional derivative.** Tangential plane and normal to the surface. Scalar field. Gradient of the scalar field.		Classroom work 15	
Topic 3.3. **Extreme functions of many variables.** Definition, necessary conditions. Sufficient conditions for the existence of a local extremum. Finding the extremums of the function of two variables. **Conditional extremum.** Level surfaces. Finding a conditional extremum. The method of Lagrange multipliers.	PLO 1, 24	**Lecture 18** Classroom work 16	16th week

Section 4. **Indefinite integrals.**			
Topic 4.1. **Antiderivative and the indefinite integral.** Some properties of an indefinite integral. Integration by substitution (change of variable).	PLO 1, 24	**Lecture 19** Classroom work 17 **Lecture 220**	17th week
Topic 4.2. **Integration.** Integrals of functions containing a quadratic trinomial. Integration by parts. Partial rational fractions and their integration. Decomposition of a rational fraction into partial fractions.	PLO 1, 24	**Lecture 21** Classroom work 18	18th week
Final lesson. Course review.		**Lecture 22**	18th week
Modular control work		Submission for verification. Presentation and defense of CGW	16 -17th week 18th week

ANALYTICAL GEOMETRY AND LINEAR ALGEBRA

Names of sections and topics	Program learning outcomes	The main tasks	
		Learning process	Deadline
Section 1. **Introduction to Linear Algebra and Matrices.**			
Topic 1.1. **Matrices and operations with them** (equality, addition, subtraction, scalar multiplication, matrix product, the power of a square matrix, transposition).	PLO 1, 24	**Lecture 1** Classroom work 1	1st week
Topic 1.2. Determinants and their properties, Laplace's expansion theorem.	PLO 1, 24	**Lecture 2** Classroom work 2	2nd week
Topic 1.3. **The inverse matrix** and its properties. Matrix equations.	PLO 1, 24	**Lecture 3** Classroom work 3	3nd week
Topic 1.4. **The rank** of a matrix, its properties, ways of finding the rank.	PLO 1, 24	**Lecture 4** Classroom work 4	4th week
Topic 1.4. **Solution of Simultaneous Linear Equations.**	PLO 1, 24	**Lecture 5** Classroom work 4	5th week

Cramer's rule, Gaussian elimination. Nonhomogeneous system of equations.			
Section 2. **Elements of vector algebra**			
Topic 2.1. Vectors and linear operations with them (addition, subtraction, scalar multiplication. Parallelism of two vectors. Unit vector.	PLO 1, 24	**Lecture 6** Classroom work 6	6th week
Topic 2.2. **Components of a vector.** Direction cosines. Magnitude of a vector. Angle between two vectors. Scalar projection of a vector onto an axis (other vector) and its properties. **Scalar product.** Scalar product for vectors given in component form.	PLO 1, 24	**Lecture 7** Classroom work 7	7th week
Topic 2.3. **Vector product**, its properties and uses. **Scalar triple product.** Finding the scalar triple product for vectors in component form.	PLO 1, 24	**Lecture 8** Classroom work 8	8th week
Topic 2.4. **Complex numbers.** Basic operations on complex numbers. Euler's formula. Formal definition of a complex number (Cartesian form), its real and imaginary parts. Operations with complex numbers. Geometric interpretation of a complex numberDe Moivre's theorem. Roots of complex numbers.	PLO 1, 24	**Lecture 9** Classroom work 9	9th week
Topic 2.5. Cartesian coordinate system in *2D* and *3D*, distance between two points, partitioning a segment in a given ratio. Polar coordinate system.	PLO 1, 24	**Lecture 10** Classroom work 10	10th week
Modular control work		Writing MCR / testing	13th week
Section 3. **Geometry on a plane and in space**			
Topic 3.1. **Lines in a plane.** Different forms of the equation of	PLO 1, 24	**Lecture 11** Classroom work 11	11th week

a line in a plane: slope-intercept form, normal vector form, Cartesian equation, parametric equation, vector equation, equation of a line passing through two points, equation of a line in a "segment form".			
Topic 3.2. **Planes.** Different forms of the equation of a plane: vector equation, Cartesian equation, equation of a plane containing three points, equation of a plane in a "segment form". Relationship between planes. The angle between two planes. The shortest distance from a point to a plane.	PLO 1, 24	**Lecture 12** Classroom work 12	12^{th} week
Topic 3.3. **Lines in *3D***. Different forms of the equation of a line in *3D*: Cartesian equation of a line, parametric equation of a line, vector equation of a line, equation of a line passing through two points, equation of a line as an intersection of two planes.	PLO 1, 24	**Lecture 13** Classroom work 13	13^{th} week
Topic 3.4. Line classification. Skew lines. Angle between two lines. Condition under which two lines are coplanar. Lines and planes combined in *3D*. Angle between a line and a plane. Intersection of a line and a plane. Different problems for lines and planes in *3D.*	PLO 1, 24	**Lecture 14** Classroom work 14	14^{th} week
Section 4. **Second Order equation equations on plane and in space**.			
Topic 4.1. **Second order curves in normal forms:** ellipse, hyperbola, and parabola.	PLO 1, 24	**Lecture 15** Classroom work 15	15^{th} week
Topic 4.2. A change of coordinate by rotation through the angle.	PLO 1, 24	**Lecture 16** Classroom work 16	16^{th} week
Topic 4.3. **Second order surfaces** in normal forms: ellipsoid,	PLO 1, 24	**Lecture 17** Classroom work 17	17^{th} week

hyperboloid, hyperboloid of two sheets, hyperbolic paraboloid, cone, cylinder, hyperbolic cylinder, parabolic cylinder.			
Topic 4.3. **Sylvester's criterion.** Eigenvalues and Eigenvectors of Matrices.	PLO 1, 24	**Lecture 18**	18th week
Modular control work		Writing MCR / testing	17th week

The discipline **"Higher Mathematics-2"** is studied in the second spring semester is represented in the next table:

Names of sections and topics	Program learning outcomes	The main tasks	
		Learning process	Deadline
Section 1. **Indefinite and the definite integrals.**			
Topic 1.1. **Integration.** Integrals of functions containing a quadratic trinomial. Integration by parts. Partial rational fractions and their integration. Decomposition of a rational fraction into partial fractions.	PLO 1, 24	**Lecture 1** **Lecture 1** Classroom work 1	1st week
Topic 1.2. **Integrals of irrational functions.** Integration of certain irrational functions by means of trigonometric substitutions. Integration of binomial differentials.	PLO 1, 24	**Lecture 3** Classroom work 2	2nd week
Topic 1.3. **The definite integral.** Statement of the problem. The lower and upper integral sums. Basic properties of the definite integral. Evaluation of integral.	PLO 1, 24	**Lecture 4** **Lecture 5** Classroom work 3	3d week
Topic 1.4. **Newton-Leibniz formula**. Changing the variable in the definite integral. Integration by parts.	PLO 1, 24	**Lecture 6** Classroom work 4	4th week

Topic 1.4. Improper Integrals. Topic 1.5. **Geometric application of the definite integral.** The Area in Rectangular Coordinates and the Area of a Curvilinear Sector in Polar Coordinates.	PLO 1, 24	**Lecture 7** **Lecture 8** Classroom work 5	5th week
Topic 1.5. **Geometric and mechanical application of the definite integral.** The Arc Length of a Curve. The Volume of a Solid of Revolution. The Surface of a Solid of Revolution. Coordinates of the Centre of Gravity.	PLO 1, 24	**Lecture 9** Classroom work 6	6th week
Section 2. **Multiple integrals and Line integrals.**			
Topic 2.1. **Multiple integrals.** Double Integrals. Calculating Double Integrals. Changing Variables in a Double Integral. The Double Integral in Polar Coordinates.	PLO 1, 24	**Lecture 10** **Lecture 11** Classroom work 7	7th week
Topic 2.2. **Application of the Double Integrals.** Calculating Areas and Volumes by Means of Double Integrals.	PLO 1, 24	**Lecture 12** Classroom work 8	8th week
Topic 2.3. **Triple Integral.** Evaluating a Triple Integral. Change of Variables in a Triple Integral. Triple integral in cylindrical coordinates. A triple integral in spherical coordinates.	PLO 1, 24	**Lecture 13** **Lecture 14** Classroom work 9	9th week
Topic 2.4. **Geometric Application of Double and Triple Integrals.** The Moment of Inertia of the Area of a Plane Figure. The Coordinates of the Centre of Gravity of the Area of a Plane Figure. The Moment of Inertia and the Coordinates of the Centre of Gravity of a Solid.	PLO 1, 24	**Lecture 15** Classroom work 10	10th week
Section 3. **Line integrals and surface integrals.**			
Topic 3.1. **Line integral of the I kind.** Evaluating a Line Integral of the I kind. Topic 3.2. **Line integral of the II**	PLO 1, 24	**Lecture 16** **Lecture 17** Classroom work 11	11th week

kind. Evaluating a Line Integral of the II kind.			
Topic 3.2. **Green's Formula.** Conditions for a Line Integral being Independent of the Path of Integration.	PLO 1, 24	**Lecture 18** Classroom work 12	12th week
Topic 3.4. **Surface Integrals I-II kind.** Evaluating Surface Integrals. Topic 3.5. **Stokes' Formula.** Topic 3.6. **Ostrogradsky's Formula.**	PLO 1, 24	**Lecture 19** **Lecture 20** Classroom work 15	13th week
Modular control work **Colloquium**		Writing MCR / testing	13th week
Topic 3.7. **Scalar and Vector Fields.** The Hamiltonian Operator and Certain Applications of It.	PLO 1, 24	**Lecture 21** Classroom work 17	14th week
Section 4. **Differential equations**.			
Topic 4.1. **Differential equations.** Statement of the Problem. First-Order Differential Equations. Equations with Separated and Separable Variables. Topic 4.2. **The initial value problem**. Homogeneous First-Order Equations. Equations Reducible to Homogeneous Equations.	PLO 1, 24	**Lecture 22** **Lecture 23** Classroom work 15	15th week
Topic 4.3. **First-Order Linear Differential equations.** Bernoulli's Equation. Exact Differential Equations.	PLO 1, 24	**Lecture 24** Classroom work 20	17th week
Topic 4.4. **Higher-order differential equations.** Some Types of Second-Order Differential Equations Reducible to First-Order Equations. Topic 4.5. **Homogeneous Linear Equations.** Definitions and General Properties. Second-	PLO 1, 24	**Lecture 25** **Lecture 26** Classroom work 21	16th week

Order Homogeneous Linear Equations with Constant Coefficients. Homogeneous Linear Equations of the n-th Order with Constant Coefficients.			
Topic 4.6. **Nonhomogeneous Linear Equations of the n-th Order with Constant Coefficients.** Nonhomogeneous Second-Order Linear Equations with Constant Coefficients. Topic 4.7. **Systems of Ordinary Differential Equations.** Systems of Linear Differential Equations with Constant Coefficients.	PLO 1, 24	**Lecture 27** Classroom work 23	17th week
Topic 4.8. **The method of variation of arbitrary constants.** Nonhomogeneous Linear Equations of the second Order with Constant Coefficients. **Final lesson. Course review.**	PLO 1, 24	**Lecture 28**	18th week
Modular control work		Writing MCR / testing	18th week

The discipline **"Higher Mathematics-3"** is studied in the third autumn semester is represented in the next table:

Names of sections and topics	Program learning outcomes	The main tasks	
		Learning process	Deadline
Section 1. **Series.**			
Topic 1.1. **Series**. Sum of a Series. Necessary Condition for Convergence of a Series. Comparing Series with Positive Terms. **Sufficient conditions for the convergence of numerical series.** D'Alembert's Test. Cauchy's Test. The Integral Test for Convergence of a Series.	PLO 1, 24	**Lecture 1** Classroom work 1 **Lecture 2** Classroom work 2	1st week
Topic 1.2. **Alternating Series.** Leibniz' Theorem. Pfus-and-Minus Series. Absolute and Contitional Convergence. Application of Leibniz' Theorem.	PLO 1, 24	**Lecture 3** Classroom work 3 Classroom work 4	2nd week

Topic 1.3. **Power Series.** Interval of Convergence. **Integration and Differentiation of Series.** Series in Powers of (x- a)	PLO 1, 24	**Lecture 4** Classroom work 5 **Lecture 5** Classroom work 7	3d week
Topic 1.4. **Taylor's Series and Maclaurin's Series.** Euler's Formula. The Binomial Series. Integrating Differential Equations by Means of Series. **Application of Taylor's Series and Maclaurin's Series.**	PLO 1, 24	**Lecture 6** Classroom work 6 Classroom work 8	4th week
Topic 1.4. **Fourier series.** Statement of the Problem. Expansions of Functions in Fourier Series. A Remark on the Expansion of a Periodic Function in a Fourier Series. Fourier Series for periodic function $f(x)$ with period 2π. Topic 1.5. **Fourier Series for Even and Odd Functions.** The Fourier Series for a Function with Period 2l. On the Expansion of a Nonperiodic Function in a Fourier Series.	PLO 1, 24	**Lecture 7** Classroom work 9 **Lecture 8** Classroom work10	5th week
Topic 1.6. **Fourier Integral.** The Fourier Integral in Complex Form. Application of the Fourier Integral.	PLO 1, 24	**Lecture 9** Classroom work 11 Classroom work 12	6th week
Section 2. **Calculus of Complex Functions.**			
Topic 2.1. **The Need For Complex Numbers.** Basic Operations with complex numbers.	PLO 1, 24	**Lecture 10** Classroom work 13 **Lecture 11** Classroom work 14	7th week
Topic 2.2. **Function of complex variables.**	PLO 1, 24	**Lecture 12** Classroom work 15 Classroom work 16	8th week
Topic 2.3. **Differentiation of complex function.** Cauchy-Riemann conditions. Analitic function.	PLO 1, 24	**Lecture 13** Classroom work 17 **Lecture 14** Classroom work 18	9th week

Topic 2.4. **Integration of complex function.** Cauchy's integral formula. Cauchy's integral formula for derivatives			
Topic 2.5. Convergence of power series.Taylor and Maclaurin series. Expand the complex function. Topic 2.6. **Taylor's theorem.** Singularities. Laurent series. Examples of Laurent series.	PLO 1, 24	Classroom work 19 **Lecture 15** Classroom work 20	10^{th} week
Topic 2.7. Poles. Residues. Application of Residues for finding meaning of integrals.	PLO 1, 24	**Lecture 16** Classroom work 21 **Lecture 17** Classroom work 22	11^{th} week
Section 3. **Operational calculus.**			
Topic 3.1. **The Laplace transform.** Original and L-Transform. The Initial Function and Its Transform. **The transform** of a function with changed scale of the independent valuable. transforms of the functions $cos\alpha t, sin\alpha t, sh\alpha t, ch\alpha t$.	PLO 1, 24	**Lecture 18** Classroom work 23 Classroom work 24	12^{th} week
Topic 3.2 Theorem of L-Transform. Table of transforms.	PLO 1, 24	**Lecture 19** Classroom work 25 **Lecture 20**	13^{th} week
Modular control work **Colloquium**		Writing MCR / testing	13^{th} week
Topic 3.3. An auxiliary equation for a given differential equation. Decomposition theorem. Investigating free oscillations.	PLO 1, 24	**Lecture 21** Classroom work 27 Classroom work 28	14^{th} week
Section 4. **Introduction to Probability Theory**.			
Topic 4.1. **Introduction to Probability Theory.** Random Experiments and Probability Models. Important definition. Geometric definition of probability.	PLO 1, 24	**Lecture 22** Classroom work 29 **Lecture 23** Classroom work 30	15^{th} week

Topic 4.2. Permutations, Combinations, and the Binomial Theorem. Topic 4.3. Product Rule. Law of Total Probability and Bayes' Rule.			
Topic 4.3. **Random Variables and Probability Distributions.** Random Variables. Probability Distribution. Expectation. Some Important Discrete and Continuous Distributions.	PLO 1, 24	Classroom work 31 **Lecture 24** Classroom work 32	16th week
Topic 4.4 **Generating Random Variables on a Computer.** Random Number Generation. The Inverse-Transform Method. Generating From Commonly Used Distributions. Topic 4.5. **Joint Distribution.** Joint Distribution and Independence. Discrete and Continuous Joint Distributions Expectation. Conditional Distribution.	PLO 1, 24	**Lecture 25** Classroom work 32 **Lecture 26** Classroom work 33	17th week
Topic 4.6. **Functions of Random Variables and Limit Theorems.** Jointly Normal Random Variables.	PLO 1, 24	**Lecture 27** Classroom work 34	18th week
Final lesson. Course review.	PLO 1, 24	**Lecture 28**	18th week
Modular control work		Writing MCR / testing	18th week

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6. Student self-study

One of the main types of semester control during the mastering of the discipline "Higher Mathematics" is the implementation of calculation and graphic work. Calculation and graphic work were performed in accordance with the requirements, within the period specified by the teacher.

The main purpose of computational and graphic work is to solve a mathematical problem using the material learned in lectures and self-study time, and practical skills acquired at seminars. The student can write a calculation and graphic work **(CGW)** only on the variant was agreed with the teacher.

Exemplary topics of calculation and graphic work (CGW)

The student receives a variant of control homework from the teacher which corresponds to the curriculum for the course.

The structure of the CGW

Calculation and graphic works consist of the following structural elements: title page, tasks on CGW, calendar plan of CGW preparation and content.

The title page is the first page of the CGW. The title page should have the information provided in the following sequence: name of the ministry, name of the university; name of the faculty; name of department; name of academic discipline; CGW topic and its variant; level of higher education; code and name of the specialty; the name of the educational and professional program; surname and name of the student, course, number of the academic group; signatures of the teacher; the result of defense; year of CGW implementation.

On the next page, there is a task on the CGW, which contains information about: the deadline for the student to complete the work, initial data for work, a list of graphics, the date of the task, a detailed schedule of CGW with deadlines for individual stages of work.

Next is the table of contents: solution of problems, which should consist of a detailed explanation, figure (if necessary), reference to the relevant theorems or formulas. Figures explaining solutions of the problems should be done on the platform Geogebra.

Registration of CGW

Registration of CGW is carried out according to DSTU 3008:2015 “Information and documentation. Reports in the field of science and technology. Structure and registration rules”.

CGW must be printed on a standard sheet of format A4 with the following requirements: left field - 30 mm, right - 15 mm, top - 20 mm, bottom -20 mm; Times New Roman font size 14 pt; line spacing - 1.5; indentation of the red line - 1.25; text alignment - width.

Each structural element of the content of the work begins with a new page. The names of the structural elements should be placed in the centre of the line without a dot at the end, without underlining, separated from the text by three line spacing. The transfer of syllables in words is not used. Figures and solutions must have headings and numbering consistent with the section number.

Assessment of CGW

CGW work is evaluated by the following criteria:

- Timeliness of execution.
- The presence of all points of the methodology of solving problems. Sequence and correctness of calculations.
- Content and completeness of the topic.
- The degree of mastery of theoretical material and methods of solving the problem.
- Consistency and correctness of calculations.
- Quality of the answer to the questions concerning the solved problems.

Policy and control

7. Policy of academic discipline (educational component)

Attending classes

Attending lectures and practical sessions is not mandatory. However, students are encouraged to attend classes, as they teach theoretical material, assess the level of its mastery during the oral survey, develop skills and abilities necessary to perform tasks in independent work.

The assessment system is focused on obtaining points for student activity, as well as performing tasks that can develop practical skills and abilities.

Control measures missed

Practical works submitted for inspection in violation of the deadline, but before the deadline for the current certification (or test / exam), are evaluated with penalty points.

Practical works submitted for inspection in violation of the deadline and after the deadline for the current certification (or test / exam) are not evaluated.

Calculation and graphic work, which is submitted for inspection in violation of the deadline, is evaluated with penalty points.

Violation of deadlines and incentive points

Encouragement points		Penalty points*	
Criterion	Weight score	Criterion	Weight score
Active participation in oral interviews	+ 1 point	Violation of terms of practical works (for each such work)	-1 point
Participation in international, national and / or other events or competitions on the subject of the discipline	+ 5 points	Untimely writing of modular control work / testing	From -2 to - 4 points (depending on the delivery date)
		Late submission of CGW for evaluate	From -2 to - 10 points (depending on the delivery date)

**If the control measure was missed for a respectful reason (illness, which is confirmed by a certificate of the established sample) - penalty points are not accrued.*

Academic integrity

The policy and principles of academic integrity are defined in Section 3 of the Code of Honor of the National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”. Details: https://kpi.ua/code

Norms of ethical behavior

Norms of ethical behavior of students and employees are defined in Section 2 of the Code of Honor of the National Technical University of Ukraine “Igor Sikorsky Kyiv Polytechnic Institute”. Details: https://kpi.ua/code

Procedure for appealing the results of control measures

Students have the opportunity to raise any issue related to the control procedure and expect it to be addressed according to predefined procedures.

The student has the right to appeal the results of control measures in accordance with the Regulations on appeals in National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute" (order №NON /128/2021 from 20.05.2021) https://osvita.kpi.ua/node/182

Inclusive education

The discipline may be taught to the majority of students with special educational needs, with the exception of persons with severe visual impairments who are not allowed to perform tasks using personal computers, laptops and / or other technical means.

Distance Learning

Distance learning is through training platform "Sikorsky".

Implementation of practical work, modular control work / testing, calculation and graphic work done during independent work of students in remote mode with the possibility of consulting with the teacher via e-mail, platform ZOOM, Skype.

Learning a foreign language

Teaching in English is carried out only for foreign students.

At the request of students, it is allowed to study the material with the help of English-language online courses on topics that correspond to the topics of specific classes.

8. Types of control and rating evaluate system of learning outcomes (Rating System of Evaluation)

Types of control and scores for each control element

№ s / n	Control measure	%	Weight score	Number	Total
1.	Classroom work	10	10	3	10
2.	Modular control work (MCW)/ Test	20	20	2	20
3.	Calculation and graphic work (CGW)	20	20	1	20
	Total				50

The results are announced to each student individually in the presence or remotely - in the Moodle system or by e-mail. In case of detection of plagiarism or establishment of non-independent performance of work, points for the control measure are not credited.

The applicant receives a positive credit score for the results of the semester, if he has a final rating for the semester of at least 60 points and has met the conditions of admission to the semester control, which are determined by the RSE (Rating System of Evaluation).

With applicants who have met all the conditions of admission to the test and have a rating of less than 60 points, as well as with those applicants who want to increase their rating, in the last scheduled lesson in the semester, the teacher conducts semester control in the form of test or interviews.

After the test, additional points can be added to the rating.

If the grade for the test is lower than the rating, a "hard" RSE is used - the previous rating of the applicant (except for points for the semester individual task) is cancelled, and he

receives a grade based on the results of the test. This option forms a responsible attitude of the applicant to the decision to perform the test, forces him to critically assess the level of his training and carefully prepare for the test.

Current control

Classroom work (Short independent written work or Google test) R1

№ s / n	Evaluation criterion	Scores
1.	Complete answer (at least 90% of the required information)	2-3
2.	Sufficiently complete answer or complete answer with minor inaccuracies (not less than 75% of the required information)	1-2
3.	Incomplete answer and minor errors (at least 60% of required information)	0-1
4.	The answer is missing or incorrect	0

Modular control work R2

№ s / n	Evaluation criterion	Scores
1.	Complete answer (at least 90% of the required information)	9-10
2.	Sufficiently complete answer or complete answer with minor inaccuracies (not less than 75% of the required information)	7-8
3.	Incomplete answer and minor errors (at least 60% of required information)	6-7
4.	The answer is missing or incorrect	0

Calculation and graphic work R3

№ s / n	Evaluation criterion	Scores
1.	Complete and timely execution of the task, error-free solution of the problem, the material is presented with knowledge of theoretical material, logically, consistently, with clarity (not less than 90% of the required information)	18-20
2.	The task is not complete enough or with some inaccuracies. Visual material is not used to the full (at least 75% of the required information).	15-18
3.	The task is not fully disclosed, partially or with some shortcomings. Visual material is insufficient (at least 60% of the required information).	11-14
4.	The task is incomplete and (or) with fundamental errors. Visual material is insufficient (less than 50% of the required information). There is no ownership of the material	0

Calendar control *(CC) -* is performed twice a semester as monitoring of the current state of compliance with syllabus requirements.

The purpose of calendar control is to improve the quality of student learning and monitor the implementation of the schedule of the educational process by students.

Criterion			The first CC	The second CC
Deadline of calendar controls			8th week	14th week
Conditions for obtaining a positive result from the calendar control	Current rating		≥ 10 points	≥ 22 points
	Execution of calculation and graphic work and Short independent written work or Google test	Problems № № 1-7, Tests	+	-
		Problems № № 8-14, Tests	-	+
	Performing MCR / testing	MCR / testing	-	+
	Colloquium	Theoretical test	+	-

Colloquium does not give points in the ranking, but must be completed by 60%

In case of detection of academic poor quality during training - the control measure is not credited.

Semester rating of students

Mandatory condition for admission to the test	Criterion
Current rating, including:	RD= ≥ 60
- implementation of MCR / testing	not less than 60% of the maximum score
- calculation and graphic work	not less than 60% of the maximum score
All classroom works and Google tests	not less than 60% of the maximum score

Student rating from the credit module “Higher mathematics-1” in the discipline “HIGHER MATHEMATICS” consists of the points that he/she receives for: **Rs=R1+R2+R3=50.**

The rating of semester is announced to each student separately in the presence or remotely (by e-mail). Also recorded in the system "Electronic Campus".

Optional conditions for admission to closure:

1. Activity in practical classes.
2. Successfully completed written tests (execution of calculation and graphic work, short independent written work) and Google test.
3. Positive result of the first attestation and the second attestation.

Semester certification of students

Written exam paper:

At the exam, the student conducts a written examination work. Each ticket consists of 1 theoretical question and 4 practical tasks. The list of theoretical questions is given by the examiner at the last discipline.

Weighted score -10. Maximum score 10 points x 5 tasks = 50 points (**R4**).

1st semester exam:

- The theoretical question - 10 points;
- Task 1 with topic Analytical geometry and linear algebra10 points;
- Task 2 of the topic Differential numbering of functions of one variable - 10 points;
- Task 3 of the topic Investigating the functions of one variable - 10 points;
- Task 4 of the topic Differential number of functions of many variables - 10 points.

2nd semester exam:

- The theoretical question - 10 points;
- Task 1 with topic Multiple integrals and Line integrals - 10 points;
- Task 2 of the topic Application of Multiple integrals and Line integrals - 10 points;
- Task 3 of the topic Differential equations. IVP -10 points;
- Task 4 of the topic Nonhomogeneous Linear Equations of the n-th Order with Constant Coefficients. IVP. Nonhomogeneous Second-Order System Linear Equations with Constant Coefficients - 10 points.

3d semester exam:

- The theoretical question - 10 points;
- Task 1 with topic Functional and Power Series - 10 points;
- Task 2 of the topic Complex Analysis- 10 points;
- Task 3 of the topic Operational calculus -10 points;
- Task 4 of the topic Introduction to Probability Theory - 10 points.

№ s / n	Criteria for evaluating each question / task test control work	Number of points for each question / task

1.	Complete answer, not less than 90% of the required information, performed in accordance with the requirements for the level of "skills" (complete, error-free solution of the problem)	9-10
2.	Sufficiently complete answer, at least 75% of the required information, performed in accordance with the requirements for the level of "skills" or there are minor inaccuracies (complete solution of the problem with minor inaccuracies)	7-8
3.	Incomplete answer, at least 60% of the required information, which is performed in accordance with the requirements for the "stereotypical" level and there are some errors (the task is performed with certain shortcomings)	6-7
4.	The answer does not meet the conditions for "satisfactory"	0

When performing a test, students are not allowed to use textbooks, abstracts, reference books, phones, smartphones, PCs, etc.

The rating obtained in the exam is added to the rating obtained during the semester:

Rs=R1+R2+R3+R4=100 points

Table of correspondence of rating points to grades on a university scale

Number points	*Assessment on the university scale*	*Possibility to receive an assessment "automatically"*
100-95	*Perfectly / Відмінно*	is
94-85	*Very good / Дуже добре*	is
84-75	*Good / Добре*	is
74-65	*Satisfactorily / Задовільно*	is
64-60	*Enough / Достатньо*	is
Less than 60	*Unsatisfactorily / Незадовільно*	-
Admission conditions are not met	*Not allowed / Не допущено*	-

9. Additional information on the discipline (educational component)

The list of questions for preparation for modular control work and test is given in Appendix 1

Distance learning through additional online courses on certain topics is allowed subject to agreement with students. If a small number of students wish to take an online course on a specific topic, studying the material with such courses is allowed, but students must complete all the tasks provided in the discipline.

The list of courses is offered by the teacher after the students have expressed a desire (because the bank of available courses is updated almost every month).

The student provides a document confirming the completion of the distance course (in the case of a full course) or provides practical tasks from the distance course and subject to an

oral interview with the teacher on the topics can receive grades for control measures provided for the studied topics (express control / test tasks, practical work).

Work program of the discipline (syllabus):

Compiled by: PhD, Senior Lecturer, **Mulyk Olena.** PhD, Senior Lecturer, **Syrotenko Anton.**
Approved by the Department of Biomedical Engineering (protocol № 13 to"25" June 2021).
Approved by the Methodical Commission of the faculty of Biomedical Engineering (Protocol №11 to "25" June 2021).

Appendix 1 to the syllabus of the discipline "Mechanics"

The list of questions for preparation for modular control work and exam

1. Convergent numerical sequences and their properties. The Limit of numerical sequence . Infinitely small and infinitely large sequences. Theorems and Definitions. Uncertainties and their types. The $\varepsilon -$ neighbourhood of a variable.
2. The number e.
3. The Limit of a function. Infinitely small and infinitely large functions. Basic theorem on Limits. The First Limit of a function $\lim_{x\to 0} \frac{sinx}{x} = 1.$
4. The Limit of a function. Infinitely small and infinitely large functions. Basic theorem on Limits. The Second Limit of a function $\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$.
5. Consequences of the Theorem of First Limit and the Second Limit. Comparing Infinitely small functions. Equivalent to infinitesimal functions.
6. Continuity of function. Break points of function and classifications discontinuities. Continuous function on the interval.
7. Definition of derivative. Geometric and physical meaning of the derivative. Continuity and the existence of the derivative at the point. The tangent and normal to the function graph.
8. Derivative of inverse, composite functions, functions given implicitly and parametrically. Derivatives of hyperbolic functions.
9. Basic theorems of differential calculus. Applications of theorems of Lagrange, Role, Cauchy.
10. Lopital`s rule.
11. Taylor's formula. Taylor's formula. Remainder of the Taylor formula in the form of Lagrange, Peano.
12. Applycation of the Taylor and MacLoren formula. MacLoren formula of basic elementary functions.
13. Investigation the behavior of functions. Increase and decrease of a function. Maxima and minima of functions. Testing a differentiable function for maximum and minimum with a first derivative. Maxima and minima of function on an interval.
14. Convex functions. Necessary and sufficient conditions of the convexity of the function on the interval. The points of inflection, sufficient conditions for the existence of the point of inflection. Asymptotes of the function graph.
15. Indefinite integral. Table of integrals. Properties of indefinite integral. Integration by substitutions. Integration by parts.
16. Indefinite integral. Integrals of function containing a quadratic trinomial. Integration of certain classes of trigonometric functions. Integration of certain irrational functions by means of trigonometric substitutions.
17. Indefinite integral. Decompositions of a rational fractions into partial fraction. Integration of rational fractions.
18. Matrices and operations with them (equality, addition, subtraction, scalar multiplication, matrix product, the power of a square matrix, transposition). Determinants and their properties, Laplace's expansion theorem.

19. The inverse matrix and its properties. Matrix equations. Methods of solving inhomogeneous system of equations: matrix method
20. The rank of a matrix, its properties, ways of finding the rank. System of equations. Inhomogeneous and homogeneous system of equations. Methods of solving inhomogeneous system of equations: Cramer's rule, Gaussian elimination. Homogeneous system of equations.
21. Vectors and linear operations with them (addition, subtraction, scalar multiplication. Parallelism of two vectors. Unit vector. Components of a vector. Direction cosines. Magnitude of a vector. Angle between two vectors. Scalar projection of a vector onto an axis (other vector) and its properties. Scalar product, its properties and applications. Scalar product for vectors given in component form.
22. Vector product, its properties and uses. Finding the vector product for two vectors given in component form. Scalar triple product, its geometrical interpretation, properties and applications. Finding the scalar triple product for vectors in component form.
23. Lines in a plane. Different forms of the equation of a line in a plane: normal vector form, Cartesian equation, parametric equation, vector equation, equation of a line passing through two points, equation of a line in a "segment form". Relationship between lines, the angle between two lines, the shortest distance from a point to a line. Different problems with lines in a plane.
24. Planes. Different forms of the equation of a plane: vector equation, Cartesian equation, equation of a plane containing three points, equation of a plane in a "segment form". Relationship between planes. The angle between two planes. The shortest distance from a point to a plane.
25. Lines in *3D*. Different forms of the equation of a line in *3D*: Cartesian equation of a line, parametric equation of a line, vector equation of a line, equation of a line passing through two points, equation of a line as an intersection of two planes.
26. Line classification. Skew lines. Angle between two lines. Condition under which two lines are coplanar. Lines and planes combined in *3D*. Angle between a line and a plane. Intersection of a line and a plane. Different problems for lines and planes in *3D*.
27. Double integral. Definitions, properties, calculations and applications. Replacement of variables in multiple integrals. Jacobian transition.
28. Triple integral. Definitions, properties, calculations and applications. Replacement of variables in multiple integrals. Jacobian transition.
29. Cylindrical and spherical coordinates. Jacobian transition.
30. Curvilinear integrals of the first kind: definition, calculation and application.
31. Curvilinear integrals of the second kind: definition, calculation and application.
32. Surface integrals of the first: definition, calculation and application.
33. Surface integrals of the second kind: definition, calculation and application.
34. Green's Formula.
35. Stokes' Formula.
36. Scalar and Vector Fields.
37. Ostrogradsky's Formula. The
38. Hamiltonian Operator and Certain Applications of It.
39. Ordinary differential equations. Problems leading to the concept of DE. Cauchy's problem (IVP). ODE applications.
40. First-order differential equations with separable variables. Homogeneous DE. Equations Reducible to Homogeneous Equations.
41. First-Order Linear Differential equations.

42. Bernoulli equations. Equations that allow lowering the order.
43. Higher-order differential equations. Some Types of Second-Order Differential Equations Reducible to First-Order Equations. IVP.
44. Homogeneous Linear Differential Equations. Definitions and General Properties. Second-Order Homogeneous Linear Equations with Constant Coefficients. Homogeneous Linear Equations of the n-th Order with Constant Coefficients.
45. Nonhomogeneous Linear Differential Equations. Definitions and General Properties. Second-Order Nonhomogeneous Linear Equations with Constant Coefficients. Homogeneous Linear Equations of the n-th Order with Constant Coefficients. IVP.
46. Free and Forced Oscillations.
47. The method of variation of arbitrary constants. Second-Order Nonhomogeneous Linear Equations with Constant Coefficients. IVP.
48. LDE systems. IVP.
49. Numerical series and functional series. Numeric series. Definition of convergent and divergent numerical series, the sum of the series. The simplest properties of numerical series. A necessary condition for the convergence of the series.
50. Numerical series. Sufficient signs of convergence of numerical series with non-negative terms: Comparison test I and II, Integral tests of Cauchy.
51. Numerical series. Sufficient signs of convergence of numerical series with non-negative terms: Radical tests of Cauchy, test of D'Alembert.
52. Alternating numerical series. Leibniz theorem. Absolute and conditional convergence of a numerical series. Properties of absolutely and conditionally convergent numerical series.
53. Functional series. Convergence of functional series. Weierstrass theorem of uniform convergence. Properties of the sum of a uniformly convergent series of continuous functions.
54. Theorems on integration and differentiation of a functional series. Power series.
55. Abel's theorem. The area and radius of convergence of the power series. Properties of power series.
56. Definition of the Taylor series and a sufficient condition for the convergence of this series.
57. Application of power series.
58. Fourier series. Orthogonal systems of functions. Orthogonality of the trigonometric system of functions. Definition of Fourier coefficients and trigonometric Fourier series.
59. Dirichlet condition of convergence of a Fourier series. Fourier series of even and odd functions.
60. A series of Fourier functions of an arbitrary period and given on a finite segment. Properties of Fourier coefficients.
61. Fourier integral. Conditions of convergence of the Fourier integral. Fourier integral of even and odd functions.
62. Theory of the function of a complex variable.
63. Complex numbers and functions of a complex variable.
64. The concept of the function of a complex variable. Limit and continuity of a function of a complex variable.
65. Elementary functions in a complex domain. Logarithmic and inverse trigonometric functions of a complex variable.
66. Differentiation and integration of the function of a complex variable. Derivative of a function of a complex variable. Differentiability of the function of a complex variable, Cauchy - Riemann conditions.

67. Integration of functions of a complex variable. Definition, properties and calculation of the integral over a complex variable.
68. Cauchy's theorem. Cauchy integral formula.
69. The Taylor and Laurent series are functions of a complex variable. Development of the analytical function in the Taylor series.
70. Development of the analytic function in the ring in the Laurent series.
71. Isolated special points of analytical function and their classification.
72. The concept of Residues of a function in its isolated singular point. Calculation of Residues.
73. Application of Residues to the calculation of definite and improper integrals.
74. Operational calculus. Laplace transform. Definition of the original function and its Laplace transform.
75. Elementary properties of Laplace transform. Image table. Laplace transform properties.
76. Finding the original.
77. Duhamel integral.
78. Application of Laplace transform to the solution of differential and integral equations, systems of differential equations.
79. Random Experiments and Probability Models. Definitions.
80. Product Rule. Law of Total Probability .
81. Bayes' Rule.
82. Random Variables and Probability Distributions.
83. Random Variables. Probability Distribution. Expectation. Some Important Discrete Distributions.
84. Random Variables. Probability Distribution. Expectation. Some Important Continuous Distributions.
85. Joint Distribution. Joint Distribution and Independence. Discrete and Continuous Joint Distributions Expectation. Conditional Distribution.
86. Functions of Random Variables and Limit Theorems.
87. Jointly Normal Random Variables.